Problem 3.41

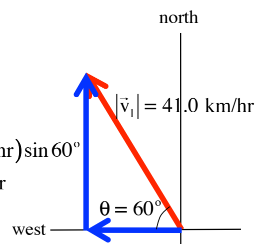
a) The polar representation of the hurricane's initial velocity is:

$$\vec{v}_1 = (41.0 \text{ km/hr}) \measuredangle 120^\circ$$

(Note: to make sense of the angle, think about the way the problem is stated.)

That vector in *unit vector notion* is most easily determined with a quick sketch (see to the right). Using the information shown there and manually inserting the appropriate negative sign, we can write:

$$|v_{1,y}| = |\vec{v}_1| \sin\theta = (41.0 \text{ km/hr}) \sin 60^\circ = 35.5 \text{ km/hr}$$



$$|v_{1,x}| = |\vec{v}_1| \cos\theta = (41.0 \text{ km/hr}) \cos 60^\circ = 20.5 \text{ km/hr}$$

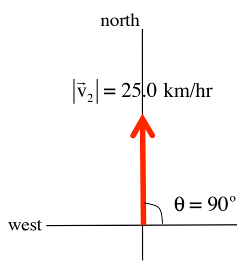
$$\vec{v}_1 = \left(20.5(-\hat{i}) + 35.5\hat{j}\right) \text{ km/hr} = \left(-20.5\hat{i} + 35.5\hat{j}\right) \text{ km/hr}$$

Note that although the negative sign technically belongs to the unit vector, it is perfectly acceptable to stick it in front of the number.

b.) The polar representation of the hurricane's velocity after change is:

$$\vec{v}_2 = (25.0 \text{ km/hr}) \measuredangle 90^\circ$$

That vector in *unit vector notion* really doesn't need a sketch, but I'm providing one in any case (see to the right). Using the information shown there, we can write:



$$\vec{v}_2 = \left(25.0\hat{j}\right) \text{ km/hr}$$

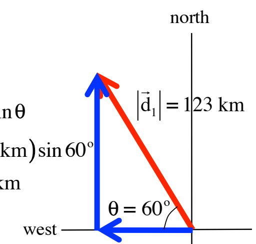
Given the fact that we started out with a sketch of each of the velocity vectors, the next part is just a copy of those graphs with distance quantities replacing the velocities.

c.) During the first *three hours*, the hurricane moved a distance of:

$$\vec{d}_1 = \vec{v}_1 \Delta t = \left[(41.0 \text{ km/hr}) \measuredangle 120^\circ\right](3.00 \text{ hr}) = (123 \text{ km}) \measuredangle 120^\circ$$

Again, that vector in *unit vector notion* is most easily determined with a quick sketch (see to the right). Using the information shown there and manually inserting the appropriate negative sign, we can write:

$$|d_{1,y}| = |\vec{d}_1| \sin\theta = (123 \text{ km}) \sin 60^\circ = 107 \text{ km}$$



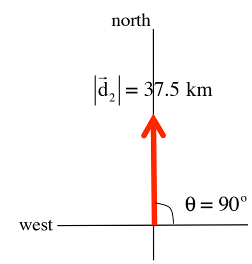
$$|d_{1,x}| = |\vec{d}_1| \cos\theta = (123 \text{ km}) \cos 60^\circ = 61.5 \text{ km}$$

$$\vec{d}_1 = \left(-61.5\hat{i} + 107\hat{j}\right) \text{ km}$$

d.) During the next *hour and a half*, the hurricane moved a distance of:

$$\vec{d}_2 = \vec{v}_2 \Delta t = \left[(25.0 \text{ km/hr}) \measuredangle 90^\circ\right](1.50 \text{ hr}) = (37.5 \text{ km}) \measuredangle 90^\circ$$

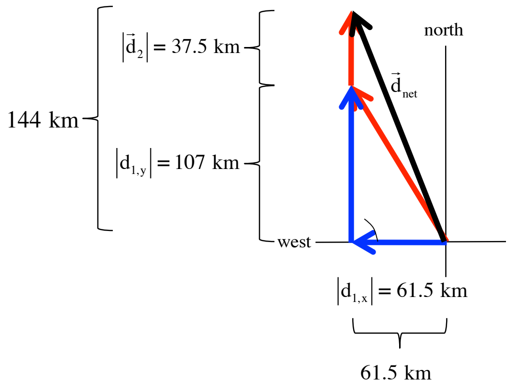
Again, that vector in *unit vector notion* really doesn't need a sketch, but you're getting you anyway (see to the right). Using the information shown there, we can write:



$$\vec{d}_2 = \left(37.5\hat{j}\right) \text{ km}$$

e.) The grand finale is the vector sum of those two distance vectors (see sketch).

Using the sketch, we can execute the vector sum in unit vector notion by inspection (sum the x-components, sum the y-components, etc.) and write:



$$\vec{d}_{net} = \left(-61.5\hat{i} + 144\hat{j}\right) \text{ km}$$

Converting this to *polar notation* yields:

$$\vec{d}_{net} = \left((-61.5)^2 + (144)^2\right)^{1/2} \measuredangle \left[\tan^{-1}\left(\frac{144}{-61.5}\right) + 180^\circ\right]$$
$$= (157 \text{ km}) \measuredangle 113^\circ$$

Minor Note: If the angle is confusing, see note below.

Soooo, apparently, the hurricane will be 157 miles from its start point after four and a half hours.

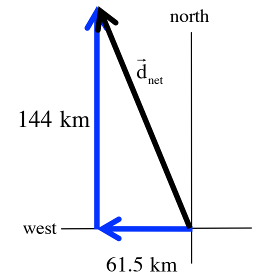
BIG NOTE about the angle:

When we took the *inverse tangent* of the y-component of the position vector divided by the x-component of the position vector, we wrote:

$$\tan^{-1}\left(\frac{144}{-61.5}\right)$$

If you actually did this and put it into a calculator, you got:

$$\tan^{-1}\left(\frac{144}{-61.5}\right) = -66.9^\circ,$$



not the correct 113 degrees. The problem resides in the fact that your calculator can't tell the difference between (144/-61.5) and (-144/61.5). The former is a fourth quadrant angle, the latter a second quadrant angle. With no guideline as to which it is, it will ALWAYS give you the fourth quadrant angle. You have to decide whether it's the right angle (this is part of the reason why a sketch usually helps with these problems), or whether you have to add 180 degrees to get the right angle. In our case, we had to add on the extra degrees.